NAG Toolbox for MATLAB

f07cu

1 Purpose

f07cu estimates the reciprocal condition number of a complex n by n tridiagonal matrix A, using the LU factorization returned by f07cr.

2 Syntax

```
[rcond, info] = f07cu(norm_p, dl, d, du, du2, ipiv, anorm, 'n', n)
```

3 Description

f07cu should be preceded by a call to f07cr, which uses Gaussian elimination with partial pivoting and row interchanges to factorize the matrix A as

$$A = PLU,$$

where P is a permutation matrix, L is unit lower triangular with at most one nonzero subdiagonal element in each column, and U is an upper triangular band matrix, with two superdiagonals. f07cu then utilizes the factorization to estimate either $\left\|A^{-1}\right\|_1$ or $\left\|A^{-1}\right\|_\infty$, from which the estimate of the reciprocal of the condition number of A, $1/\kappa(A)$ is computed as either

$$1/\kappa_1(A) = 1/\left(\|A\|_1\left\|A^{-1}\right\|_1\right)$$

or

$$1/\kappa_\infty(A) = 1/\left(\|A\|_\infty\left\|A^{-1}\right\|_\infty\right).$$

$1/\kappa(A)$ is returned, rather than $\kappa(A)$, since when A is singular $\kappa(A)$ is infinite.

Note that $\kappa_\infty(A) = \kappa_1\left(A^{\mathrm{T}}\right)$.

4 References

Higham N J 2002 *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

5 Parameters

5.1 Compulsory Input Parameters

1: **norm_p – string**

Specifies the norm to be used to estimate $\kappa(A)$.

norm_p = '1' or 'O'

Estimate $\kappa_1(A)$.

norm_p = 'I'

Estimate $\kappa_\infty(A)$.

Constraint: **norm_p** = '1', 'O' or 'I'.

2: **dl**$(*)$ **– complex array**

Note: the dimension of the array **dl** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n-1)$ multipliers that define the matrix L of the LU factorization of A.

3: **d**$(*)$ **– complex array**

Note: the dimension of the array **d** must be at least $\max(1, \mathbf{n})$.

Must contain the n diagonal elements of the upper triangular matrix U from the LU factorization of A.

4: **du**$(*)$ **– complex array**

Note: the dimension of the array **du** must be at least $\max(1, \mathbf{n} - 1)$.

Must contain the $(n - 1)$ elements of the first superdiagonal of U.

5: **du2**$(*)$ **– complex array**

Note: the dimension of the array **du2** must be at least $\max(1, \mathbf{n} - 2)$.

Must contain the $(n - 2)$ elements of the second superdiagonal of U.

6: **ipiv**$(*)$ **– int32 array**

Note: the dimension of the array **ipiv** must be at least $\max(1, \mathbf{n})$.

Must contain the n pivot indices that define the permutation matrix P. At the ith step, row i of the matrix was interchanged with row **ipiv**(i), and **ipiv**(i) must always be either i or $(i + 1)$, **ipiv**$(i) = i$ indicating that a row interchange was not performed.

7: **anorm – double scalar**

If **norm_p** = '1' or 'O', **anorm** must contain $\|A\|_1$.

If **norm_p** = 'I', **anorm** must contain $\|A\|_\infty$.

anorm must be computed either before calling f07cr, or else from a copy of the original matrix A.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The dimension of the array **d** The dimension of the array **ipiv**.

n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

work

5.4 Output Parameters

1: **rcond – double scalar**

Contains an estimate of the reciprocal condition number.

2: **info – int32 scalar**

info = 0 unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If **info** $= -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **norm_p**, 2: **n**, 3: **dl**, 4: **d**, 5: **du**, 6: **du2**, 7: **ipiv**, 8: **anorm**, 9: **rcond**, 10: **work**, 11: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

7 Accuracy

In practice the condition number estimator is very reliable, but it can underestimate the true condition number; see Section 15.3 of Higham 2002 for further details.

8 Further Comments

The condition number estimation typically requires between four and five solves and never more than eleven solves, following the factorization. The total number of floating-point operations required to perform a solve is proportional to n.

The real analogue of this function is f07cg.

9 Example

```
norm_p = '1-norm';
dl = [complex(-0.78, -0.26);
     complex(0.162, -0.4859999999999999);
     complex(-0.04516923076923077, -0.0009538461538460446);
     complex(-0.3978553846153843, -0.05620307692307711)];
d = [complex(1, -2);
     complex(1, +1);
     complex(2, -3);
     complex(1, +1);
     complex(-1.339863692307691, +0.2875264615384604)];
du = [complex(-1.3, +1.3);
     complex(-1.3, +3.3);
     complex(-0.3, +4.3);
     complex(-3.3, +1.3)];
du2 = [complex(2, +1);
     complex(-1, +1);
     complex(1, -1)];
ipiv = [int32(2);
     int32(3);
     int32(4);
     int32(5);
     int32(5)];
anorm = 9.388448823157418;
[rcond, info] = f07cu(norm_p, dl, d, du, du2, ipiv, anorm)
```

```
rcond =
    0.0054
info =
           0
```